69 $\underline{A}$ $|r| < 1$

$$\sum_{n=0}^{\infty} (x-1)^n = \int 1 + (x-1) + (x-1)^2 \dots$$

$|x-1| < 1$

$a = 1 \quad r = (x-1)$

$-1 < x-1 < 1$

$0 < x < 2$

$\frac{a}{1-r}$

70 E $\frac{1}{1-(x-1)} = \frac{1}{2-x}$

71 $\int_0^x \frac{1}{2-t}\,dx = -\ln|2-t| \Big|_0^x = -\ln|2-x| - -\ln 2$

$-\ln|x-2| + \ln 2$

$-(\ln|x-2| - \ln 2)$

$-\ln\left(\frac{|x-2|}{2}\right)$ D

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63. $\int \frac{1}{x} = \int \frac{1}{1+(x-1)} = \int 1 - (x-1) + (x-1)^2 - (x-1)^3 \dots$

$a = 1 \quad r = -(x-1)$

$$\ln x = x - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3} - \frac{(x-1)^4}{4} \dots + c$$

$x = 1 \quad 0 = 1 + c$

$\ln 1 = 0 \quad c = -1$

$$\ln x = (x-1) - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3} - \dots$$

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$\frac{8}{1-.6} - 4$

$a = 8 \quad r = .6$

17. $\sum_{n=0}^{\infty} \sin^n\left(\frac{\pi}{4} + n\pi\right) = 1 + \sin\left(\frac{3\pi}{4}\right) + \sin^2\left(\frac{\pi}{4} + 2\pi\right)$

$= 1 - \frac{\sqrt{2}}{2} + \frac{1}{2} \quad \cdots$

$a = 1 \quad r = -\frac{\sqrt{2}}{2}$

$\frac{a}{1-r} = \frac{1}{1+\frac{\sqrt{2}}{2}} \frac{2}{2} = \frac{2}{(1+\sqrt{2})} \frac{(1-\sqrt{2})}{(1-\sqrt{2})}$

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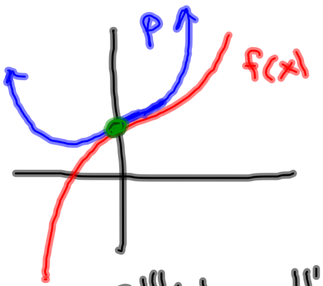
9.2 Taylor series (Maclaurin series)

given $f(x)$, find its power series

$f(x) \approx p(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 \cdots$



$f(0) = p(0) = a_0$ ✓

$f'(0) = p'(0)$

$f'(0) = a_1$ ✓

$p'(x) = a_1 + 2a_2 x + 3a_3 x^2 \cdots$

$f''(0) = p''(0)$

$p''(x) = 2a_2 + 3 \cdot 2 a_3 x + \cdots$

$f''(0) = 2a_2$

$\frac{f''(0)}{2} = a_2$ ✓

$p'''(x) = 3 \cdot 2 a_3 + \cdots$

$f'''(0) = p'''(0)$

$f'''(0) = 3 \cdot 2 a_3$

$\frac{f'''(0)}{3 \cdot 2 \cdot 1} = a_3$ ✓

$a_4 = \frac{f^4(0)}{4!}$

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$$f(x) \approx \frac{f(0)}{0!} + \frac{f'(0)}{1!}x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \frac{f^4(0)}{4!}x^4 \cdots$$

$$\sum_{n=0}^{\infty} \frac{f^n(0)}{n!}x^n \quad \text{for } x \text{ close to } 0$$

$0! = 1$

Maclaurin series for $f(x)$

$f(x) = e^x$ $\quad f(0) = 1$

$f'(x) = e^x$ $\quad f'(0) = 1$

$f''(x) = e^x$ $\quad f''(0) = 1$

$f^3(x) = e^x$ $\quad \vdots \quad \vdots$

$\vdots \quad \vdots$

$$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \frac{x^4}{4!} \cdots$$

let $x = 1$

$$e \approx 1 + 1 + \frac{1}{2} + \frac{1}{3!} + \frac{1}{4!} \cdots$$

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$f(x) = \sin x$

$f'(x) = \cos x$

$f''(x) = -\sin x$

$f'''(x) = -\cos x$

$f^4(x) = \sin x$

$f^5(x) = \cos x$

$\vdots \quad \vdots$

$f(0) = 0$

$f'(0) = 1$

$f''(0) = 0$

$f'''(0) = -1$

$f^4(0) = 0$

$\vdots$

1

0

-1

0

1

$\vdots$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} \cdots$$

$$\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$$

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